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Combining Abstract Domains for Software Model Checking

Bachelor's Thesis

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HALLGATÓI NYILATKOZAT

Alulírott *Bajkai Viktória Dorina*, szigorló hallgató kijelentem, hogy ezt a szakdolgozatot meg nem engedett segítség nélkül, saját magam készítettem, csak a megadott forrásokat (szakirodalom, eszközök stb.) használtam fel. Minden olyan részt, melyet szó szerint, vagy azonos értelemben, de átfogalmazva más forrásból átvettem, egyértelműen, a forrás megadásával megjelöltem.

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Budapest, 2018. december 7.

Bajkai Viktória Dorina
hallgató

Kivonat

Mindennapi életünket egyre jobban meghatározzák a szoftverrendszerek. Ezek sokszor biztonságkritikusak (pl. autonóm járművek, erőművek), tehát helyes működésük garantálása kiemelten fontos feladat. Ennek egyik eszköze a formális verifikáció, ami a hibák jelenlétét és a helyes működést is képes matematikailag precíz módon bizonyítani. Az egyik legelterjedtebb formális verifikációs módszer a modellellenőrzés, amely a program összes lehetséges állapotát és átmenetét (azaz állapotterét) szisztematikusan megvizsgálja. A módszer egyik hátránya viszont a nagy számítási igénye, ami gyakran megakadályozza használatát valós szoftvereken.

Az ellenpélda-alapú absztrakciófinomítás (angolul Counterexample-Guided Abstraction Refinement, CEGAR) egy olyan kiegészítő technika, melynek segítségével a modellellenőrzés hatékonyabbá tehető. Működése során a CEGAR iteratívan hozza létre és finomítja az ellenőrzendő probléma egy absztrakcióját. Az irodalomban több különböző absztrakciós megközelítés létezik, például az explicit változók módszere, illetve a predikátumabsztrakció. Előbbi a programnak csak a verifikáció céljából releváns változóit tartja nyilván, míg az utóbbi konkrét értékek helyett matematikai kifejezések teljesülését vizsgálja. Korábbi eredmények alapján megfigyelhető, hogy különböző absztrakciós módszerek különböző típusú szoftvereken működnek hatékonyabban. Ebből kifolyólag létrejöttek úgynevezett szorzat absztrakciók, amik többféle módszert kombinálnak egy algoritmusban.

Munkám során eltérő stratégiák alapján kombináltuk az explicit változókat predikátumokkal. Megközelítésünk lényege, hogy a már felderített absztrakt állapottérből kinyert információk figyelembe vételével a további felderítést és ellenőrzést hatékonyabbá teszi. Ezeket az új stratégiákat a Theta nevű nyílt forráskódú verifikációs keretrendszerben implementáltuk. Ennek segítségével szoftverrendszerek széles skáláján tudtuk lefuttatni méréseinket, többek között ipari vezérlő (PLC) kódokon. Összevetettük a különböző stratégiák előnyeit és hátrányait, és a már létező módszerekkel is összehasonlítottuk őket. Az eredményeink azt mutatják, hogy az új módszereink hatékonyan tudják kombinálni a meglévő algoritmusok előnyeit.

Abstract

Software systems are controlling devices that surround us in our everyday life. Many of these systems are safety-critical (e.g., autonomous vehicles, power plants), thus ensuring their correct operation is gaining increasing importance. Formal verification techniques can both reveal errors and give guarantees on correctness with a sound mathematical basis. One of the most widely used formal verification approaches is model checking, which systematically examines all possible states and transitions (i.e., the state space) of the software. However, a major drawback of model checking is its high computational complexity, often preventing its application on real-life software.

Counterexample-guided abstraction refinement (CEGAR) is a supplementary technique, making model checking more efficient in practice. CEGAR works by iteratively constructing and refining abstractions in a given abstract domain. There are several existing domains, such as explicit-values, which only track a relevant subset of program variables and predicates, which use logical formulas instead of concrete values. Observations show that different abstract domains are more suitable for different kinds of software systems. Therefore, so-called product domains have also emerged that combine different domains into a single algorithm.

In this work, we develop and examine various strategies to combine the explicit-value domain with predicates. Our approaches use different information from the already explored abstract state space to guide further exploration more efficiently. We implement our new strategies on top of Theta, an open source verification framework. This allows us to perform an experiment with a wide range of software systems including industrial PLC codes. We evaluate the strengths and weaknesses of the different approaches and we also compare them to existing methods. Our experiments show that the new strategies can form efficient combinations of the existing algorithms.

Chapter 1

Introduction

Nowadays our reliance on safety-critical software systems is rapidly increasing. Therefore, there is a growing need for reliable proofs of their correct behaviour, since a failure can lead to serious damages. A promising approach for giving such proofs is formal software verification. Formal verification provides a sound mathematical basis to prove the correct operation of the programs with mathematical precision. A widely used formal verification method is *model checking*, which analyses the possible states and transitions (i.e., the state space) of the software for every possible input and checks whether certain properties are satisfied. In our current work, we are checking for assertion failures, but in general a wide variety of properties can be examined, including overflows, null pointers and indexing out of bounds. The advantage of model checking is that it can not only reveal faults, but prove their absence as well. However, a major drawback is that systematically examining every possible state and transition for each input is too expensive computationally. Even for relatively simple programs the state space can be large or even infinite, which is called the "state space explosion problem". Various techniques have been developed in the past decades to overcome this problem, including symbolic methods, bounded model checking and abstraction. In our work, we use the supplementary technique *counterexample-guided abstraction refinement* (CEGAR).

CEGAR is a widely used software model checking algorithm, which uses abstraction to represent the state space in a more compact way. Abstraction means hiding certain details about the program. However this does not only yield a smaller state space, but we also lose information about the program. The abstraction usually over-approximates the original program. This means, that if no erroneous behaviour (i.e., counterexample) can be found in the abstraction, then the original program is also safe. However, losing information can also lead to finding a counterexample in the abstraction, that does not exist originally. That means, that the abstraction has to be refined to exclude the spurious counterexample. CEGAR usually starts with a coarse initial abstraction of the program and automatically finds the proper level of abstraction by a series of refinement steps.

CEGAR can work with different abstract domains, such as explicit-value analysis and predicate abstraction. Explicit-value analysis operates by tracking values of only a subset of the program's variables, while predicate abstraction focuses on tracking certain facts (predicates) about the variables. However, different abstract domains are more suitable for different kinds of software. Combinations of abstract domains, called product abstractions can unify the strengths of the different approaches. However, a key challenge is to find the proper way of combining them.

In our work, we develop a product abstraction algorithm, which combines explicit-value analysis and predicate abstraction. We try to focus on the advantages of both algorithms to propose five different strategies. These approaches use different information from the abstract state space (e.g., single states, paths, or all states) and different state enumeration strategies to combine explicit-value analysis with predicate abstraction efficiently.

In order to evaluate and compare these strategies, we implement them in Theta, an open source verification framework. We evaluate the performance of the new algorithms on multiple types of programs, including industrial programmable logic controller (PLC) codes from CERN, and several types of programs from the Competition on Software Verification (SV-Comp). We also compare the new strategies with the existing explicit-value analysis and predicate abstraction methods. The results show that our new algorithms can combine the advantages and outperform existing methods relying only on a single domain.

Chapter 2

Background

In this chapter we present the background of product abstraction-based software model checking. We describe programs using Control Flow Automata (Section 2.1), a formal representation based on graphs and first order logic formulas. Then we introduce abstraction and the CEGAR approach (Section 2.2), which is a widely used technique for software verification.

2.1 Control flow automata

Programs can be described in various ways. Humans usually work with source code as it is readable and understandable. Computers on the other hand mostly work with a compiled binary, which can be executed. For verification purposes some formal representation is required, which allows mathematical reasoning. A widely used formal representation is the Control Flow Automata (CFA). It is also called a *model* of the program. A CFA is a graph-based representation annotated with first order logic (FOL) [15] formulas to describe the operations of the program. Given a domain D, let FOL^D denote formulas of that domain. For example, $FOL^{\mathbb{B}}$ denotes Boolean formulas, e.g. $x = y \wedge y > 5$.

While FOL is undecidable in general [15], in practice *satisfiability modulo theory* (SMT) solvers [4, 22] can efficiently reason about FOL formulas in many theories that appear in programs (e.g., integer arithmetic, arrays). In our work, we also use SMT solvers to reason about the satisfiability of formulas.

Definition 1 (Control Flow Automata). A control flow automata [9] is a tuple $CFA = (V, L, l_0, E)$ where

- $V = \{v_1, v_2, \ldots, v_n\}$ is a set of *program variables* with domains $D_1, D_2, \ldots, D_n$,
- $L = \{l_1, l_2, \ldots, l_k\}$ is a set of *program locations* representing the program counter,
- $l_0 \in L$ is the *initial location*, i.e., the entry point of the program,
- $E \subseteq L \times Ops \times L$ is a set of *directed edges* between locations, representing the operations, which are executed when we go from the source location to the target in the program. ▪

Currently we are working with Boolean and (mathematical) integer variables, but in general bit-precise representation and managing floating-point arithmetic, arrays, etc. are also possible.

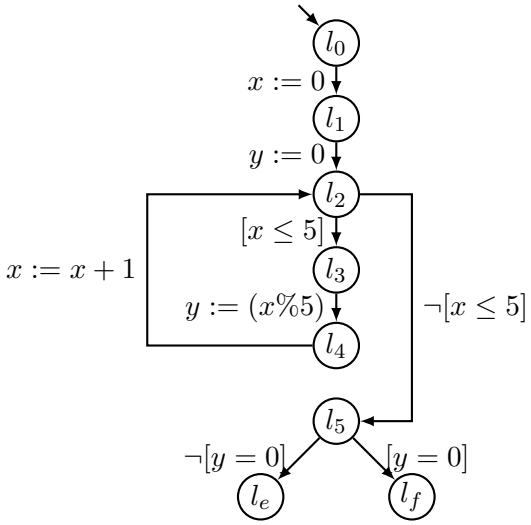
The operations $op \in Ops$ can be *assumptions, assignments* or *havocs*. Assumptions are Boolean expressions (also called predicates) denoted by $[\varphi]$ where $\varphi \in FOL^{\mathbb{B}}$. If there is an edge between two locations with an assumption, the program can take a transition to the target location if the predicate holds in the source location.

Assignments are in the form $v_i := \psi$, where $v_i \in V$ and $\psi \in FOL^{D_i}$. After this operation, v_i will be assigned the result of evaluating ψ in the target location. All other variables will have the same value as in the source location.

Havocs have the form *havoc* v_i, where at the target location v_i will be assigned a random value from its domain. Havoc operations can be used to model non-deterministic values, for example an input provided by the user or the return value of an unknown external function.

```
1 int x = 0;
2 int y = 0;
3 while (x <= 5) {
4   y = x % 5;
5   x++;
6 }
7 assert(y == 0);
```

(a) Example program.



(b) CFA representation.

Figure 2.1: Simple program and its corresponding CFA.

Example 1. *We can see an example program in Figure 2.1a. The program has two variables,* x *and* y*. In the program,* x *counts up to 5 assigning* $x\%5$ *to* y *in every cycle. At the end there is an assertion which checks whether the value of* y *is 0. In Figure 2.1b, the corresponding CFA can be seen. The initial location is* l_0*, which is the entry of the program. The first two lines are encoded by path* $l_0 \to l_1 \to l_2$*, where we arrive at the head of the loop. If the condition holds, the program enters the body of the loop by moving to* l_3*. Then the program moves to* l_4 *with an assignment and returns to* l_2*, the head of the loop, incrementing* x*. If the loop condition does not hold any more, the program moves to* l_5*, where the assertion is evaluated. The assertion is treated as a special case: If the condition holds, the program arrives to its end, a final location, which is* l_f *(in this example, but it is possible that there is more code after the assertion, and in that case, the program moves forward as usual). Otherwise it reaches* l_e*, a special error location to signal the assertion failure, which will be described later.*

State space. The actual state of the program can be described by the program counter (the actual location) and its data (the values of the variables). Therefore, the set of possible states for a program is $C = L \times D_1 \times ... \times D_n$. A *concrete state* $c \in C$ is $c = (l, d_1, ..., d_n)$, which is a location and a value for each variable from its domain.

In the CFA model, each variable is uninitialized at the beginning. Therefore, any state $c = (l_0, d_1, \ldots, d_n)$ with the initial location l_0 is considered to be an *initial state* of the program.

A *transition* $c \xrightarrow{\text{op}} c'$ between two concrete states $c = (l, d_1, \ldots, d_n)$ and $c' = (l', d'_1, \ldots, d'_n)$ exists, if there is an edge $(l, op, l') \in E$ between the locations of the two states with the semantics of the operation *op*.

- If *op* is an assumption $[\varphi]$, then φ has to hold for $d_1, \ldots, d_n$ and the values do not change, i.e., $d_k = d'_k$ for each k.
- If *op* is an assignment $d_i := \psi$, d'_i will be equal to the result of evaluating ψ, while the other variables will remain unchanged, i.e., $d'_k = d_k$ for each $k \neq i$.
- If *op* is a havoc over d_i, then d'_i can take any value, but the other variables must be unchanged, i.e., $d'_k = d_k$ for each $k \neq i$.

Note that the semantics described above means that if an edge with an assumption operation starts from the source state, then there may be 0 or 1 target state. If the operation is an assignment, there will be exactly 1 target state and in case of a havoc, the number of the target states corresponds to the size of the havoced variable's domain.

A *concrete path* $c_1 \xrightarrow{op_1} c_2 \xrightarrow{op_2} \ldots \xrightarrow{op_{n-1}} c_n$ is an alternating sequence of concrete states and transitions.

The states, the initial state and the transitions together define the *state space* of the program.

Software model checking. During software verification, a wide variety of properties can be verified, including overflow, null pointers and indexing out of bounds [6]. In our work, we focus on verifying *assertion* failures in the input programs. These assertions are represented in CFA with a choice: if the condition of the assertion holds, the program moves forward to the next location, but if it does not hold, it goes to a distinguished *error location* denoted by l_e (see Example 1).

The purpose of *software model checking* [20] is to check if a program state with the error location $(l_e, d_1, \ldots, d_n)$ is reachable with any valuation of the variables, i.e., whether an assertion failure can occur. Note that this is different than just checking if the error location is reachable in the graph of the CFA. The semantics of the operations also need to be considered. From now on, if we refer to the reachability of l_e, we mean reaching some state in the state space, which has l_e as its location.

A CFA is called *safe* if l_e is not reachable, otherwise it is *unsafe*. If the CFA is unsafe, a path $c_1 \xrightarrow{op_1} c_2 \xrightarrow{op_2} \ldots \xrightarrow{op_{n-1}} c_n$ leading to the state $c_n = (l_e, \ldots)$ with the error location is called a *counterexample*, as it is a witness for the assertion failure. Such counterexamples are important because they help the program developer to identify the source of the problem.

Software model checking is a very complex problem, because if we want to prove that the error location is unreachable, we have to explore the whole state space of the program, which can be very large or even infinite. For example, if a program has 100 locations and three 64 bit integer variables, the number of possible states is $100 \cdot 2^{64} \cdot 2^{64} \cdot 2^{64} \approx 6.2 \cdot 10^{59}$. This problem is often called the "state space explosion". To overcome this limitation of software model checking, various techniques have been developed in the past decades,

including symbolic methods [16], partial order reduction [31], bounded model checking [14], modular verification [30] and abstraction [17, 25]. In this thesis we focus on CEGAR [18], an abstraction-based algorithm, which we present in the next section.

2.2 Counterexample-guided abstraction refinement

Abstraction is a general method to reduce the complexity of a task by hiding certain details. In the context of software model checking this yields a smaller abstract state space compared to the original (concrete) state space, mitigating the problem of state space explosion. Intuitively, a single abstract state can represent multiple (or even infinite) concrete states [17]. Applying abstraction also means that we lose information, which can lead to incorrect results. However, if we use an *over-approximating* abstraction [17], the incorrect results are only one sided. This means, that if the error location is not reachable in the abstract state space, it is also not reachable in the original state space, i.e., the original program is safe. On the other hand, we might find a counterexample (path leading to the error location) in the abstract state space, which does not exists in the original program. Such counterexamples are called *spurious* and in this case a more precise abstraction is required.

The granularity of the abstraction (i.e., the amount of information hidden) is called the *precision* [9]. For example, a possible abstraction is to omit certain variables from the software and treating them as if they could take any value from their domain, represented by a single unknown value. In this case, the precision can be controlled by the amount of variables omitted: fewer omitted variables give more precise abstraction (but possibly larger state space).

Counterexample-Guided Abstraction Refinement (CEGAR) [18] is a widely used technique in software model checking [23, 27, 7, 29], which starts with an initially coarse abstraction to avoid state space explosion. Then it applies refinements iteratively until all spurious counterexamples are eliminated (proving safety) or a real counterexample is found (proving the program to be unsafe).

The steps of a typical CEGAR algorithm [34] can be seen in Figure 2.2. The two main components are the *abstractor* and the *refiner*, whose detailed behaviour will be presented in Section 2.2.1 and Section 2.2.2 respectively. The first step is to build the initial abstraction from the initial (usually coarse) precision, which is done by the abstractor. When a counterexample is found, it is passed to the refiner. If there are no counterexamples, the model is safe due to the over-approximating [17] nature of abstraction. In the next step, the refiner checks whether the counterexample is feasible. If it is feasible, the original model is unsafe. Otherwise, we have a spurious counterexample and the precision of the abstraction is refined, allowing the abstractor to build a more precise (but potentially larger) abstract state space in the next iteration. This process is iterated until there are no abstract counterexamples or a feasible one is found.

2.2.1 Abstraction

An abstract *domain* is defined by the set of *abstract states* S, the *coverage relation* $\sqsubseteq$, the set of possible *precisions* Π and the *transfer function* T [9]. Informally, the abstract domain controls the *kind* of information that is hidden to obtain abstract states and the precision defines the *amount* of information to be hidden. As mentioned previously, a single abstract state can represent any number of concrete states. The coverage relation

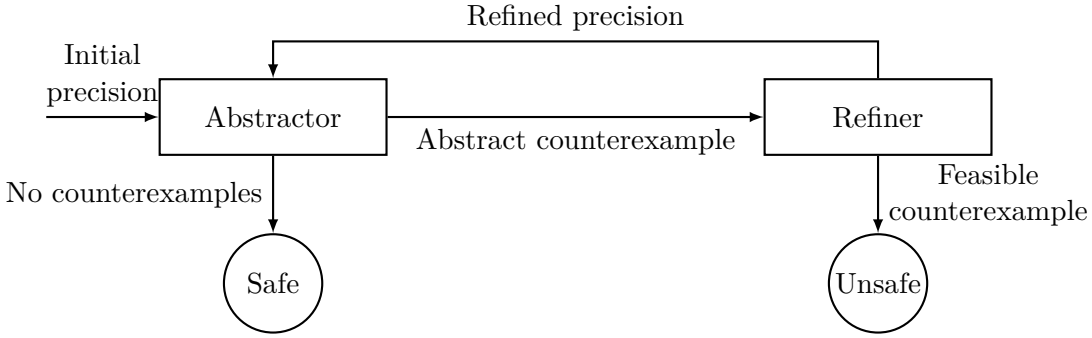


Figure 2.2: CEGAR algorithm.

$s \sqsubseteq s'$ holds for two abstract states $s, s' \in S$, if s' represents all the states that s does. Intuitively, this means that if we already processed s', we can skip s since if the error location is reachable from s, it would have already been reached from s'. The transfer function defines the successor (transition) relation between abstract states.

The abstractor builds the abstract state space for a given domain (also called an *abstract reachability graph*, ARG [8]) using the components above.

Definition 2 (Abstract reachability graph). Formally, an abstract reachability graph is a tuple $ARG = (S, A, C)$ where

- S is a set of abstract states from the domain,
- $A \subseteq S \times Ops \times S$ is a set of edges defined by the transfer function T between abstract states, labelled with operations.
- $C \subseteq S \times S$ is a set of covering edges defined by the covering relation $\sqsubseteq$. ▪

The abstractor starts with the initial abstract state, which corresponds to the initial location l_0 and has usually no information as no variable is initialized. Then it maintains a queue for the unprocessed states. As long as the queue is not empty, it picks an abstract state and checks if it can be covered with some already explored state. If yes, it adds the covering edge and leaves the state. Otherwise, it uses the transfer function to calculate its successors. The abstractor stops if there are no more states in the queue or if a state with the error location l_e is found.

In our work, we use three different abstract domains, namely *predicate abstraction* [25], *explicit-value analysis* [7] and their combination, the *product abstraction* [10]. We formalize these domains in the rest of this section.

Explicit-value analysis. Explicit-value analysis is a widely used abstraction method [19, 7, 33]. It tries to reduce the size of the state space by tracking only a subset of the program variables. Usually only a few or no variables are tracked initially and the set of tracked variables is iteratively expanded during the refinement phase. The motivation behind explicit-value analysis is that proving safety (or finding a counterexample) may only depend on a small subset of the program variables.

The set of all possible precisions is $\Pi_e = 2^V$, i.e., all possible subsets of the variables. For example, if the program's variables are $\{x, y, z\}$, then $\Pi_e = \{\{\}, \{x\}, \{y\}, \{z\}, \{x, y\}, \ldots, \{x, y, z\}\}$. A precision $\pi_e \in \Pi_e$ simply defines the

subset of the tracked variables ($\pi_e \subseteq V$), which is also called the set of explicitly tracked variables.

If a variable is not tracked (or unknown), its value is represented by a special *top element* $\top$, meaning that it can take any value from its domain. Given a variable v_i with its domain D_i, let $D_i^\top = D_i \cup \{\top\}$ represent its extension with the top element.

Abstract states $S_e = L \times D_1^\top \times \ldots \times D_n^\top$ track the location and the value of each variable in π_e or $\top$ for variables outside π_e. For example, if there are three variables $V = \{x, y, z\}$ and the precision is $\pi_e = \{x, y\}$, the state $(l_1, 0, 10, \top)$ means that the program is at location l_1, where $x = 0$, $y = 10$ and z is not tracked. Note that it is also possible for a tracked variable to be unknown ($\top$), for example if x is tracked but z is not, the assignment $x := z$ will make $x = \top$.

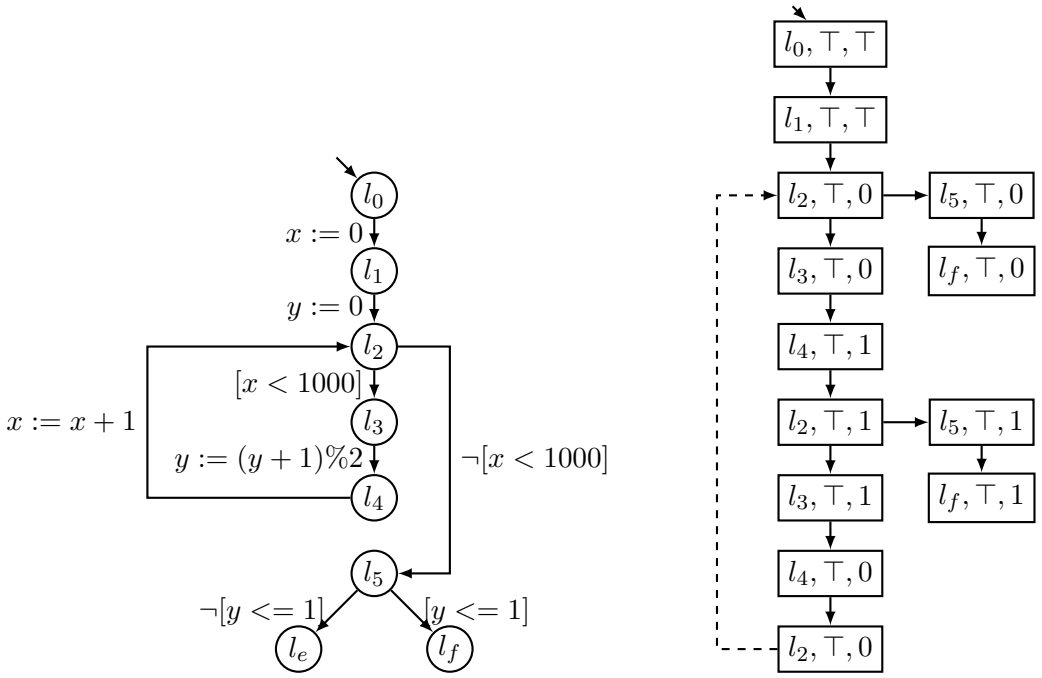
For each state we calculate its successors with the transfer function $T_e : S_e \times Ops \times \Pi_e \to 2^{S_e}$, which yields a set of successor states for a given state, operation and precision. The values of the tracked variables of the new state depend on the type of operation given to the transfer function.

- If the operation is an assumption $[\varphi]$, we have to check whether it can be evaluated over the source state. If it evaluates to true or can not be evaluated, a successor state is created, where the values of the tracked variables are unchanged. For example, if the assumption is $[x > 5]$ and x is $\top$, the expression cannot be evaluated, therefore we add a successor state, since x can take any value.
- If the operation is an assignment $v_i := \psi$, a successor can be created. If v_i is not in the set of explicitly tracked variables, the new state's variables are left unchanged. Otherwise, v_i' is assigned the result of evaluating ψ, or if it cannot be evaluated, it is assigned $\top$, while the other variables are unchanged. For example, let the precision be $\pi_e = \{x, y\}$, the source state be $s_e = (l_1, 2, 0, \top)$ and the operation be $op = (x := z + 1)$. Since z is not tracked, the operation cannot be evaluated, therefore the successor is $(l_1, \top, 0, \top)$.
- If the operation is a havoc, a successor state is created. If the havoced variable is not tracked, the new state's variables are left unchanged. Otherwise, the havoced variable is assigned $\top$ and the other values do not change.

The coverage relation $s \sqsubseteq s'$ holds between two states $s, s' \in S$, if the locations are equal ($l = l'$) and the values of s' are broader than the values of s. This means that if a v_i variable has a value d_i in s, then it must have d_i or $\top$ in s'. For example, $(l_0, 1, 2) \sqsubseteq (l_0, \top, 2)$, but $(l_0, 1, 2) \not\sqsubseteq (l_0, 3, \top)$. Intuitively, if we have a covered state, we do not have to explore the paths starting from this state as they would lead to the same states as the transitions of the covering state.

Example 2. *Consider the CFA in Figure 2.3a. It alternates the variable y between* 0 *and* 1 *while x counts up to 1000. At the end it checks whether* $y \leq 1$. *In Figure 2.3b the corresponding ARG with* $\pi_e = \{y\}$ *can be seen. In this case we only track y, because tracking x while it counts to 1000 would create too many states to explore. The initial state is* $(l_0, \top, \top)$, *since x is not tracked (because of this, the value of x will be* $\top$ *throughout the whole example) and y is not initialized. The first two transitions set the variables of the program and we arrive to state* $(l_2, \top, 0)$. *Here we are at the head of a loop, whose condition* $(x < 1000)$ *cannot be evaluated since we do not track* x. *Thus the program has to explore both possibilities. If we do not enter the loop, we move to* $(l_5, \top, 0)$, *where we arrived at the evaluation of the assertion. Since we know that* $y = 0$, *we can only*

move to the final location. Otherwise, if we enter the loop, we move to $(l_3, \top, 0)$*. The next transition is an assignment, which we can evaluate because we know the value of* y*. Therefore, we arrive at* $(l_4, \top, 1)$*. In the next step* x *is incremented and we move back to* l_2*, but this time the value of* y *is* 1*. Here we can enter the loop again reaching* $(l_3, \top, 1)$ *and then* $(l_4, \top, 0)$ *and* $(l_2, \top, 0)$*. This is a state where we have been before, so we do not have to explore again. We mark this fact with the dashed covering edge. From* $(l_2, \top, 1)$*, we can also move forward to the end of the loop,* $(l_5, \top, 1)$*, where we reach the assertion again. Since we know the value of* y*, we can evaluate the assertion, therefore we only reach the final state* $(l_f, \top, 1)$*. At this stage, there are no more states left to explore, and since we did not reach the error location, the program is safe. This example shows that we can successfully verify a program with only tracking one variable.*



(a) Example CFA. **(b)** Corresponding ARG.

Figure 2.3: CFA an its corresponding ARG created with $\pi_e = \{y\}$.

Predicate abstraction. In predicate abstraction [25, 18, 2], the values of the variables are not tracked explicitly, but instead certain facts are stored about them. For example, we do not track the exact values of a variable x, but only the fact whether $x < 5$ holds. These facts are called *predicates*, which are Boolean FOL formulas. In case of a too coarse abstraction, refinement is performed by extending the set of tracked predicates.

The set of precisions is $\Pi_p = 2^{FOL^{\mathbb{B}}}$, i.e., all possible subsets of all Boolean formulas. A precision $\pi_p \in \Pi_p$ is a set of Boolean FOL formulas ($\pi_p \subseteq FOL^{\mathbb{B}}$) that are currently tracked.

Abstract states $S_p = L \times 2^{FOL^B}$ are also subsets of predicates with the additional location component. An abstract state $s_p \in S_p$ for the actual precision π_p can contain each predicate of π_p ponated or negated. It is also possible that a predicate does not occur in an abstract state. Then, it represents both cases (it can hold or not). For example, if we

have the precision $\pi_p = \{x < 5, y \geq 7\}$, the abstract state $(l_1, \neg(x < 5))$ means that the program is at location l_1, where $x < 5$ does not hold and $y \geq 7$ can both hold or not.

An abstract state represents all concrete states for which the predicates evaluate to true. For example, $(l_1, \neg(x < 5))$ represents states with location l_1, $x = \{4, 3, 2, 1, 0, -1, \ldots\}$ and any value for the other variables.

For each state we calculate its successors with the transfer function $T_p : S_p \times Ops \times \Pi_p \to 2^{S_p}$. It works based on a source state, an operation and a target precision. The successor states will include predicates based on the operations.

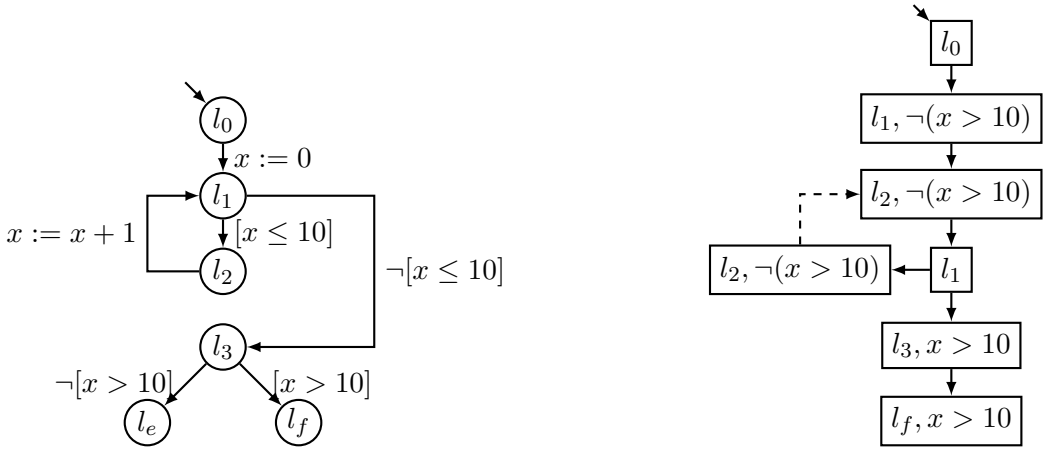
- If the operation is an assumption $[\varphi]$, we check whether the conjunction of the predicates of the source state and φ is feasible. If yes, a successor state is created. The successor state will have all predicates from the precision that are implied by the source state and the assumption[1]. Similarly, if their negation is implied, the successor state will include the negated version. For example, if the source state has the predicate $x \geq 0$ and the assumption is $[x < 0]$, there will be no successor state. On the other hand, if the assumption is $[x \leq 0]$, then a successor state is possible. If there is a predicate $x \neq 0$ in the precision, the successor state can include its negation, as $x \geq 0$ and $x \leq 0$ together imply that $\neg(x \neq 0)$.
- If the operation is an assignment $v_i := \psi$, a new state is created. Similarly to the assumptions, we check whether the predicates of the source state and the assignment imply predicates in the precision or their negated form. For example, let the precision be $\pi_p = \{(x < 5), (y \geq x)\}$, the abstract state be $s_p = (l_1, (x < 5), \neg(y \geq x))$ and the operation be $op = (x := x + 1)$. Incrementing x means that $(x < 5)$ might hold or not, since the value of x can reach 5. On the other hand, it also means that the predicate $\neg(y \geq x)$ holds, because increasing x does not change the fact, that y is less than x. Therefore, the new state is $(l_2, \neg(y \geq x))$.
- If the operation is a havoc, a successor state is created. If the havoced variable appears in a predicate, that predicate will be lost in the target state. Other predicates are left unchanged.

The covering relation $s \sqsubseteq s'$ holds for two states if the locations are equal ($l = l'$) and the predicates of s imply the predicates of s'. For example, $(x < 4)$ implies that $(x < 5)$, hence, if we already explored all states from $(x < 5)$, then it already covers all possibilities from $(x < 4)$ as well.

Example 3. *An example for predicate abstraction can be seen in Figure 2.4. In Figure 2.4a, there is an example CFA with a variable x, which counts up to 11 and checks whether its value is greater than 10. In Figure 2.4b, an abstract state space created with precision $\pi_p = \{(x > 10)\}$ can be seen. In the initial state l_0, x has not been initialized yet, therefore we cannot decide if the predicate is true or false. In the next step x is assigned the value 0, hence the $\neg(x > 10)$ predicate holds at l_1. The program then arrives to a selection, and because of the predicate, the program can only move to l_2. Since no assignment happened, the $\neg(x > 10)$ predicate still holds. In the next step x is incremented while returning to l_1, therefore we can not evaluate the predicate any more. That is why the program can move to both ways from here. If $x \leq 10$, it reaches the state $(l_2, \neg(x > 10))$, which is covered as it already appeared. Otherwise it moves to $(l_3, x > 10)$. At l_3, the program arrives to the evaluation of the assertion. It can only go to the final location*

[1] We check such implications using an SMT solver.

from here, because the predicate $x > 10$ holds. Therefore the CFA is safe. This example demonstrates that we could prove the safety of the program by tracking a single predicate instead of concrete values.



(a) Example CFA. **(b)** Corresponding ARG.

Figure 2.4: CFA an its corresponding ARG created with the precision $\pi_p = \{(x > 10)\}$.

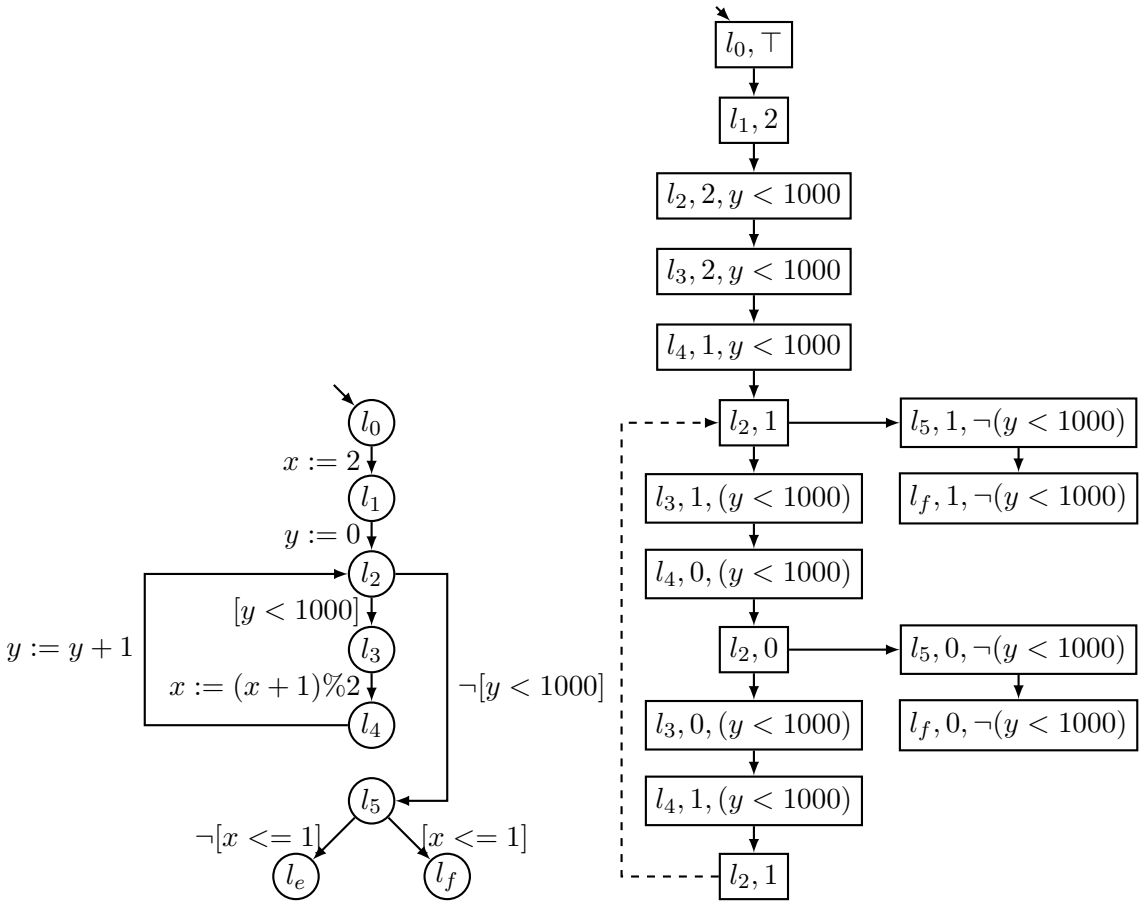
The predicate abstraction presented in our work is the Cartesian abstraction, which labels the states with the ponated or negated form of the predicates. Note that other abstraction strategies exist too, like the Boolean predicate abstraction [2], which handles any combination of predicates. Our product algorithms can work with these strategies as well.

Product abstraction. Product abstractions [10] combine different abstract domains. In our case we use a combination of explicit-value analysis and predicate abstraction. The precision is $\Pi = \Pi_p \times \Pi_e$, the combination of the two precisions. An abstract state $S = L \times 2^{FOL^{\mathbb{B}}} \times D_1^{\top} \times \ldots \times D_n^{\top}$ consists of a location and of predicates and explicitly tracked values at the same time. The transfer function $T : S \times Ops \times \Pi \to 2^S$ gets a product state $s = (l, s_p, s_e)$, an operation op, a precision $\pi = (\pi_p, \pi_e)$ and calculates $T_p((l, s_p), op, \pi_p) \times T_e((l, s_e), op, \pi_e)$, that is the Descartes product of the successor predicate states and successor explicit states[2]. A state $(l, s_p, s_e) \in S$ is covered by another state $(l', s'_p, s'_e) \in S$ if both components are covered, i.e., $(l, s_p) \sqsubseteq (l', s'_p)$ and $(l, s_e) \sqsubseteq (l', s'_e)$.

The main research question in product abstraction is how to select which variable gets tracked explicitly and which one gets predicates. To address this question, we developed multiple strategies to select between domains for the variables, which we present in Chapter 3.

Example 4. *Consider the example CFA in Figure 2.5a. It has two variables, an integer x and y. The program enters a loop and y starts to count up to 1000, while the value of x alternates between 0 and 1. At the end, it checks whether $x \leq 1$. It is very similar to the example CFA in Figure 2.3a, but the difference is that x is assigned 2 in the first step. Therefore, we have to make sure that the program enters the loop at least once to avoid arriving at the assertion while $x = 2$. This means that tracking x explicitly is not sufficient, we also have to add for example the predicate $(y < 1000)$ to the precision. In Figure 2.5b*

[2]In our current work, there is an optimization which sorts out the infeasible states. E.g. the $(x = 1)$ explicit state with the $(x < 0)$ predicate will be removed.



(a) Example CFA. **(b)** Corresponding ARG.

Figure 2.5: CFA and its corresponding ARG created with $\pi = \{x, (y < 1000)\}$.

the corresponding abstract state space can be seen, created with precisions $\pi_e = \{x\}$ *and* $\pi_p = \{(y < 1000)\}$*. Therefore an abstract state consists of the name of the location, the value of* x *and the predicate* $(y < 1000)$*. The initial state is* $(l_0, \top, \{\})$ *since the CFA starts at* l_0 *and no variables are initialized. We do not know the value of* x *and cannot evaluate the predicate. After initializing the variables in path* $l_0 \to l_1 \to l_2$*, the program arrives at* $(l_2, 2, (y < 1000))$*. Here it enters the loop, since we know that* $(y < 1000)$*. In the next step, the program evaluates the assignment and moves to* $(l_4, 1, (y < 1000))$*. The next transition increments* y *and takes the program back to the head of the loop. Since the value of* y *changed, we cannot evaluate the predicate any more, therefore the program arrives at* $(l_2, 1, \{\})$*, where it can enter the loop again, or jump to the end, to the evaluation of the assertion. If if jumps to the assertion, after evaluating it, the program arrives to final location* $(l_f, 1, \neg(y < 1000))$*. If it enters the loop, the program moves to* $(l_3, 1, (y < 1000))$*, then with the evaluation of the assignment, it arrives to* $(l_4, 0, (y < 1000))$*. After incrementing* y *the program moves to the head of the loop again. If it enters, after the assignment and the incrementation steps, the program arrives to the* $(l_2, 1, \{\})$ *state. Since we have been at this state before, we do not have to explore it again. From the head of the loop, the program can also move to the assertion,* $(l_5, 0, \neg(y < 1000))$*. Since the value of* x *is 0, we can only reach the final location,* $(l_f, 1, \neg(y < 1000))$*. At this point, there are no more states left to explore, and since we did not reach the error location, the program is safe.*

2.2.2 Refinement

The refiner's task is to check whether the abstract counterexample is feasible, and if not, it has to find a new precision to avoid the same spurious counterexample in the next iteration. It checks the counterexample by translating the alternating sequence of states and actions into FOL formulas and giving them to an SMT solver [35, 13]. If the SMT solver can find a satisfying assignment, it corresponds to a concrete counterexample.

Otherwise, the counterexample is spurious and the refiner extracts the reason of infeasibility using for example interpolation [28, 36, 26], unsat cores [27], weakest preconditions [3] or strongest postconditions [1]. This will yield new variables or new predicates that should be tracked. In our work, we treat the refinement as a black-box, which gives a new precision based on the path (counterexample) that should be joined to the old precision. We use an interpolation based method [26], which is described for transition systems, but can also work for CFA.

- In explicit-value analysis, the refiner $R_e \colon PATH \mapsto \Pi_e$ gives new variables to be tracked.
- In predicate abstraction the refiner $R_p \colon PATH \mapsto \Pi_p$ gives new predicates to be tracked.
- In the product abstraction the refiner $R \colon PATH \mapsto \Pi_p \times \Pi_e$ can call both components R_e and R_p and decide which variables and predicates to include. We developed different strategies for this decision, which we present in Section 3.

Chapter 3

Product abstraction strategies

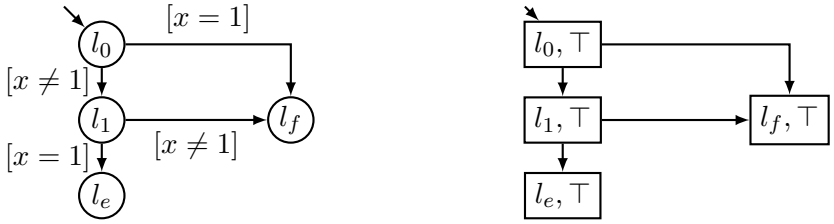
In this chapter we present five different approaches for combining explicit-value analysis and predicate abstraction into a product abstraction. When combining the two different abstractions, a very important question arises: What should be the new precision when refining the abstraction? The new precision can include new predicates or we can extend the set of explicitly tracked variables, or we can also combine these two methods by picking a predicate for some variable but adding another one to the explicitly tracked set.

The main principle is the same for all five strategies: at first, we add a new variable to the explicitly tracked variable set. Our motivation was that handling predicate formulas is more expensive computationally (e.g., checking implications in the transfer function and in the coverage relation). However, the abstract state space might start growing quickly if a variable has a large number of different values and some problems might even not be decidable due to unknown ($\top$) values.

Consider the example CFA in Figure 3.1a, which first checks if $x \neq 1$ and then if $x = 1$ (which obviously cannot be possible). Suppose, that no variables are tracked initially. In this case, the error location is trivially reachable and the refiner extends the precision by adding x. In Figure 3.1b, the corresponding ARG created with explicit-value analysis can be seen. Since x is not initialized, the initial state is $(l_0, \top)$. Then we check the condition $x \neq 1$. Since x is unknown, the condition can both hold and not. If it does not hold, the program terminates in the final location l_f. However, if it holds we proceed to l_1, where x is still $\top$. Then we check the condition $x = 1$, which can again hold or not, due to x being unknown. This way, the program can still reach the error location. Since there are no other variables to be tracked, the program cannot be verified (with explicit-value analysis).

A solution for this can be that instead of assigning $\top$ to x at l_1, we start to list all the values x can be assigned. It is an effective strategy, if the variable can only have a couple of different values, since knowing the exact values yields more information than a $\top$ value. For example, if the assumption is $0 < x < 5$, then x can only have 4 different values and creating four successor states is still more effective than using a $\top$ value or using predicates. However, in this case this does not solve our problem, since $(x \neq 1)$ means that x can have an infinite number of different values, and this leads to state space explosion.

Our key idea is to introduce a limit k for the number of possible values for each tracked variable. When we cannot evaluate an expression in the transfer function (e.g., $x \neq 1$), we start to enumerate the possible values instead of using a $\top$ value. However, we count the number of different values for each tracked variable and if it is greater than k, we stop



(a) Simple CFA. **(b)** Corresponding ARG with $\pi_e : \{x\}$.

Figure 3.1: Simple CFA and its corresponding ARG with explicit-value analysis.

enumerating and we discard the variable from the explicitly tracked set. We also add this variable to a special *dropouts* set, which is passed over to the refiner.

The strategy of the product refiner R is the following. It calculates both new variables π'_e using R_e and new predicates π'_p using R_p. All variables included in the *dropouts* set are removed from π'_e, since we do not want to track them again. Instead, we (only) keep predicates from π'_p that have a removed variable, other predicates are discarded. In other words, we first always add a variable to the explicit precision. If it is later removed during the transfer function due to the limit, we do not add it again, but rather add predicates containing that variable.

This way we can (1) avoid working unnecessarily with computationally expensive predicates and (2) we can solve problems that need explicit enumeration instead of a top value, while still avoiding state space explosion.

There are multiple ways to count the different values of the variables. In this chapter we present three different approaches. These limit the number of successor states based on a single state, or a path, or the whole ARG. The path- and ARG-based strategies also work if we use $\top$ values when an expression cannot be evaluated, since the limit can still be reached on the whole path or in the ARG. In case of the state-based strategy, this method is useless, because using $\top$ values instead of enumerating would only result in 0 or 1 successor states, therefore we would never reach the limit. That is why we implemented two different algorithms for the path- and ARG-based strategies, one which enumerates all possible states, and one which uses $\top$ values instead. This means that in total we have five different strategies to combine explicit-value analysis and predicate abstraction.

3.1 Limit number of successors based on a single state

In the first strategy (Algorithm 1), we count the different values of the tracked variables when enumerating successors for a given state. We start by initializing the successor states S'_e as an empty set. We also set a restart flag which will be used later. Then we start enumerating the successor states and we examine the values of the explicitly tracked variables. If the number of the different values of a variable in the successor states exceeds k, we add it to the *dropouts* set. We also remove it from π_e, and we set the flag that we should restart the enumeration, since the precision changed (at least one variable was dropped). If there are no more successor states to list and no variable was removed, we do not need to restart and we can return the successor states S'_e.

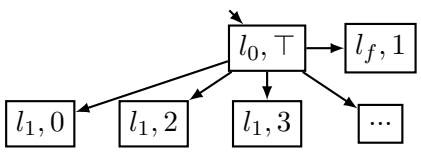
Example 5. *Consider the example CFA in Figure 3.1a again. If we use the state-based product abstraction, the variable x is added to the set of explicitly tracked variables as pre-*

Algorithm 1: State-based transfer function $T_S(s_e, op, \pi_e, k)$.

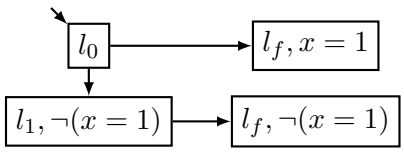
Input : s_e: source state
op: operation
π_e: precision
k: bound for successors
Output: $S'_e \subseteq 2^{S_e}$: set of successor states

```
do
    S'_e ← {}
    restart ← false
    while new successor state s'_e exists ∧¬restart do
        S'_e ← S'_e ∪ {s'_e}
        foreach v_i ∈ π_e do
            if number of different values for v_i in S'_e greater than k then
                dropouts ← dropouts ∪ {v_i}
                π_e ← π_e \ {v_i}
                restart ← true
            end
        end
    end
while restart;
return S'_e
```

viously ($\pi_e = \{x\}$). The corresponding ARG for this precision can be seen in Figure 3.2a. The program starts at state $(l_0, \top)$, from where it can go to two different directions. Taking the assumption $[x = 1]$, it arrives at state $(l_f, 1)$ since $x = 1$ is the only possible value satisfying the formula. Otherwise, the program moves to l_1, where it starts to list the possible values for $[x \neq 1]$. There are infinitely many different values, but when we exceed k, the algorithm stops. It removes x from the set of explicitly tracked variables and restarts the enumeration. However, now x is not tracked, so it proceeds to $(l_1, \top)$ without enumerating values and then eventually reaches the error location l_e similarly to Figure 3.1b. The refiner will not add x again since it is included in the dropouts set. Instead, it adds some predicate, e.g., $x = 1$ to the precision π_p. Figure 3.2b shows the ARG created with the new precision. From l_0, the program can arrive to final location $(l_f, x = 1)$ where the predicate is true or move to $(l_1, \neg(x = 1))$ where the negation of the predicate holds. At this point, the predicates keep track that $x \neq 1$ so the algorithm can only proceed to $(l_f, \neg(x = 1))$, where we reached the final location again. Since there are no more states to explore and the algorithm did not reach the error location, the program is safe.



(a) ARG created with precisions $\pi_e : \{x\}$ and $\pi_p : \{\}$.



(b) ARG created with precisions $\pi_e : \{\}$ and $\pi_p : \{(x = 1)\}$.

Figure 3.2: ARGs created with the state-based strategy.

3.2 Limit number of values on a path

The previous strategy only counted different values for the successors of a single state. However, multiple values can occur in other ways as well. For example, if the program includes a loop counting to a large number, then the loop counter i will have a single successor $i + 1$ for each state. However, if we consider the whole path, many different values will start to accumulate: $1, 2, 3, \ldots$.

This example motivated our next strategy, where we examine the number of values of the tracked variables on the path leading to a state when we calculate its successors. Algorithm 2 presents the procedure for this strategy, which is similar to the state-based. When we start to enumerate the new successors of a state, we check each new state's ancestors. If the number of different values of an explicitly tracked variable in the new state and its ancestors reach k, we add this variable to the *dropouts* set, remove it from the precision and set the restart flag.

Algorithm 2: Path-based transfer function $T_a(s_e, op, \pi_e, k)$.

Input : s_e: source state
op: operation
π_e: precision
k: bound for successors
Output: $S'_e \subseteq 2^{S_e}$: set of successor states

```
1  do
2      S'_e ← {}
3      restart ← false
4      while new successor state s'_e exists ∧¬restart do
5          S'_e ← S'_e ∪ {s'_e}
6          foreach v_i ∈ π_e do
7              if different values for v_i in ancestors of s_e and states in S'_e is greater
                than k then
8                  dropouts ← dropouts ∪ {v_i}
9                  π_e ← π_e \ {v_i}
10                 restart ← true
11             end
12         end
13     end
14 while restart;
15 return S'_e
```

Example 6. *Consider the example CFA in Figure 3.3. The program's only variable x counts to 1001, then examines whether its value is greater than 1000. Using path-based product abstraction, x is first added to π_e. When creating the ARG, we arrive at the head of the loop from the initial location. If the program stays in the loop, we get a path, where the value of x is increasing continuously, therefore the number of different values can reach the limit (the corresponding ARG can be seen in Figure 3.4a). When we exceed the limit, we remove x from the set of explicitly tracked variables and instead treat it as a top value. This way the error location can be reached. The refiner will not include x in π_e again, but rather add a predicate, e.g., $x > 1000$ to the precision π_p. The ARG created with the new precision can be seen in Figure 3.4b. The program starts at (l_0), where the predicate cannot be evaluated. After initializing x, it arrives at $(l_1, \neg(x > 1000))$. Because*

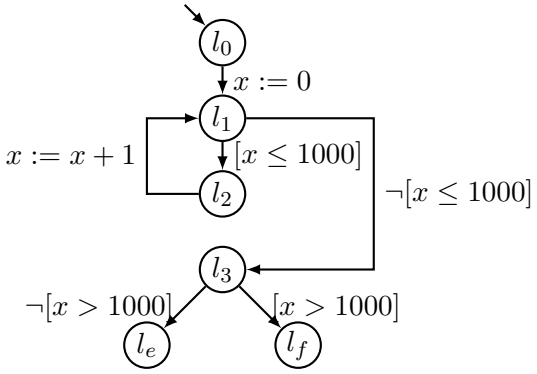
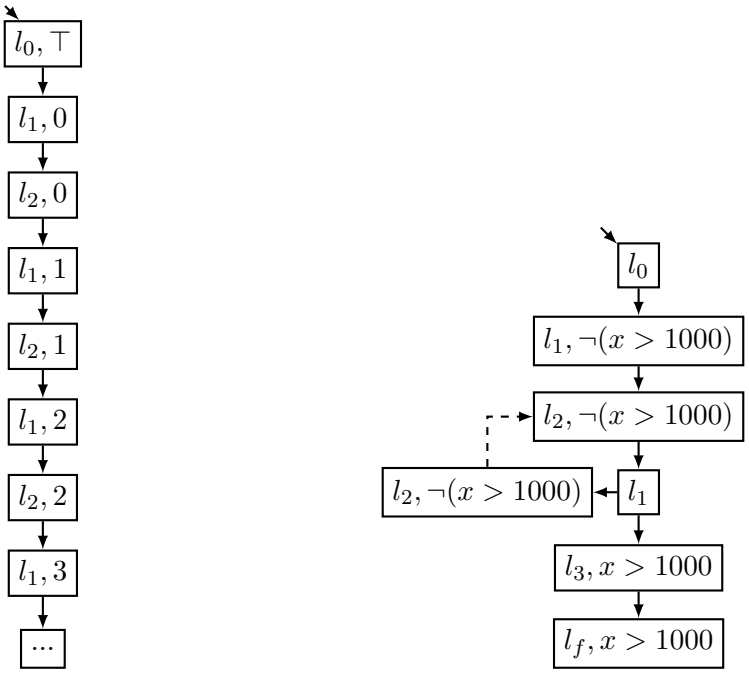


Figure 3.3: Example CFA.

of the predicate, the program moves to $(l_2, \neg(x > 1000))$*. In the next step, the value of* x *is increased, therefore we cannot evaluate the predicate any more, and arrive to* (l_1)*. The program is at the head of the loop again, but now it can go to two different directions. If it enters the loop, it arrives to* $(l_2, \neg(x > 1000))$ *again. Otherwise it moves to* $(l_3, x > 1000)$*, from where it arrives at the final location,* $(l_f, x > 1000)$*. Since there are no more states to explore and the algorithm did not reach the error location, the program is safe. The advantage of the path-based approach is that we did not have to explore all* 1001 *values for* x*.*



(a) ARG created with $\pi : \{x\}$. **(b)** ARG created with $\pi : \{(x > 1000)\}$.

Figure 3.4: AGRs created with the path-based strategy.

As mentioned previously, we proposed another algorithm for the path-based strategy, which does not enumerate all the possible states when an expression cannot be evaluated, but uses the $\top$ value instead. This way the limit can still be reached on the whole path leading to a state. That is why we check the number of values of the explicitly tracked variables before enumerating the successors of a state. Algorithm 3 presents the procedure. First we count the number of different values for each variable v_i in the ancestors of s_e

(including s_e). If a variable's number of values exceeded the limit k, we add this variable to the *dropouts* set, and remove it from the precision. Then we simply use the original transfer function T_e with the new precision to calculate the successors. This way when there are multiple values in the successors, we use the $\top$ value instead of enumerating them.

Algorithm 3: Path-based transfer function without enumeration $T_P(s_e, op, \pi_e, k)$.

Input : s_e: source state
op: operation
π_e: target precision
k: bound for successors
Output: $S'_e \subseteq 2^{S_e}$: set of successor states

```
1 foreach v_i ∈ π_e do
2 |   if number of different values for v_i in ancestors of s_e > k then
3 |   |   dropouts ← dropouts ∪ {v_i}
4 |   |   π_e ← π_e \ {v_i}
5 |   end
6 end
7 S'_e ← T_e(s_e, op, π_e)
8 return S'_e
```

3.3 Limit number of values in ARG

Our third strategy examines the number of the different values in the whole ARG. It generalizes the previous algorithms: it examines the variables in the successor states and also in the previous states through the whole ARG. Algorithm 4 presents the procedure, similar to the previous strategies, except that here we count the number of different values in the whole ARG. Note that in the implementation we use a cache, so that we do not have to traverse the whole ARG at every calculation. Whenever a new state is calculated or a variable is removed, the cache is updated.

For the ARG-based strategy, we also have another algorithm where we use $\top$ values instead of enumerating all possible states, which is presented in Algorithm 5. It works by the same principle as the path-based non-enumerating algorithm. If a variable's number of values exceeded the limit k, we add this variable to the *dropouts* set, remove it from the precision and then we use the original transfer function T_e to calculate the successors, but now some variables might have been removed.

3.4 Related work

The combination of different abstract domains have been studied in the literature before. The dynamic precision adjustment approach [10] for the explicit and predicate domains is similar to our ARG-based strategy. The main difference is that our algorithms can also enumerate states for a formula, while the dynamic precision adjustment method keeps top values.

Refinement selection [11] focuses on the different refinements returned by the explicit and predicate refiners. Various metrics are defined to compare possible refinements and pick

the "better" one. In contrast, our method always tries the explicit refinement, but then switches to predicate if needed.

Algorithm 4: ARG-based transfer function $T_a(s_e, op, \pi_e, k)$.

Input : s_e: source state
op: operation
π_e: precision
k: bound for successors
Output: $S'_e \subseteq 2^{S_e}$: set of successor states

```
do
    S'_e ← {}
    restart ← false
    while new successor state s'_e exists ∧¬restart do
        S'_e ← S'_e ∪ {s'_e}
        foreach v_i ∈ π_e do
            if number of different values for v_i in the ARG and in the states of S'_e
              is greater thank k then
                dropouts ← dropouts ∪ {v_i}
                π_e ← π_e \ {v_i}
                restart ← true
            end
        end
    end
while restart;
return S'_e
```

Algorithm 5: ARG-based transfer function without enumeration $T_P(s_e, op, \pi_e, k)$.

Input : s_e: source state
op: operation
π_e: target precision
k: bound for successors
Output: $S'_e \subseteq 2^{S_e}$: set of successor states

```
foreach v_i ∈ π_e do
    if number of different values for v_i in the ARG > k then
        dropouts ← dropouts ∪ {v_i}
        π_e ← π_e \ {v_i}
    end
end
S'_e ← T_e(s_e, op, π_e)
return S'_e
```

Chapter 4

Evaluation

This chapter presents our implementation of the five product abstraction-based strategies and their evaluation, including a comparison to explicit-value analysis and predicate abstraction. We ran measurements for every strategy with multiple different limits to examine which limit is the most effective for the different algorithms. We then compare the strategies with each other and the two basic algorithms, explicit-value analysis and predicate abstraction.

4.1 Implementation

We implemented the algorithms based on the open source[1] Theta framework [34], which is a modular and configurable model checking framework developed at the Budapest University of Technology and Economics. The explicit-value analysis and predicate abstraction algorithms, and the abstractor and refiner components were already included in Theta [26, 34]. Furthermore, Theta uses Z3 [21] as an SMT solver.

We had to implement the transfer functions for the product abstraction-based strategies and since the refiner has to know which explicitly tracked variables had been removed, we had to modify the refiner as well. We implemented these components in a Gradle[2] Java project, where Theta is imported as a Gradle plug-in.

We also implemented a runnable tool which is deployed in a jar file named `prodanalysis.jar` to run the algorithms with command line arguments. These arguments are given by the following flags.

- `model`: This is a mandatory argument, the path of the CFA file[3] to be checked.
- `domain`: This is the abstract domain to use. Its possible values are `EXPL` for explicit-value analysis, `PRED` for predicate abstraction and `PROD2` for product abstraction. It is also a mandatory argument.
- `prodstrategy`: This is the strategy to run product abstraction with. Possible values: `STATE` for state-based (Section 3.1), `PATH` for path-based (Section 3.2) and `ARG` for ARG-based (Section 3.3).

[1] `https://github.com/FTSRG/theta`

[2] `https://gradle.org/`

[3] Theta supports a simple textual description for the CFA with frontends in different languages, which will be mentioned later.

- `limit`: This is the limit k for product abstraction. It is an optional parameter with a default value of 5.
- `useTop`: This is a Boolean flag for the path- and ARG-based algorithms. When it is set to *true*, these strategies use the transfer function which includes the $\top$ values instead of enumerating possible states for expressions that cannot be evaluated. It is an optional parameter with a default value of *false*.

For example, the following call checks `example.cfa` with the state-based product abstraction strategy, where $k = 2$: `java -jar prodanalysis.jar --model example.cfa --domain PROD2 --prodstrategy STATE --limit 2`.

4.2 Measurement configuration

We ran the measurements on a 64 bit Ubuntu 16.04 operating system, with the tool RunExec fron the BenchExec suite [12]. RunExec ensures highly accurate results, since it measures the actual time spent on the CPU and also takes various side-effects into consideration (e.g., memory swapping). BenchExec is also used at the Competition on Software Verification (SV-Comp) [5].

We evaluated 430 input programs from four different sources and categories: plc, eca, locks, ssh. The 90 programs in plc are industrial programmable logic controller (PLC) codes from CERN [24], while the other three categories contain C programs[4] coming from the Competition on Software Verification (SV-Comp) [5, 6]. The category eca contains 180 programs, which describe large event-driven systems, where the events are represented with non-deterministic variables. The category locks contains 143 programs with small locking mechanisms described with non-deterministic integers and if-then-else constructs. The programs in category ssh describe 17 large server-client systems.

We evaluated these programs with 22 different configurations: PRED, EXPL and PROD2 represents the predicate abstraction, the explicit-value analysis and the product abstraction respectively. Behind PROD2, STATE, PATH and ARG represents the different product abstraction strategies presented before. The letter E means the strategy enumerates the possible values while T means that it uses top values. Finally, the number corresponds to the current limit (1, 2, 8 and 32). Thus we have $5 \cdot 4 = 20$ product strategies, and predicate and explicit abstractions giving two more.

We ran every configuration on every model, yielding 9460 measurements. We enforced a time limit of 180 seconds and a memory limit of 4 GB. With this time limit, 5828 measurements terminated successfully. We also checked that the result of the algorithms (safe/unsafe) always matches to the expected result, increasing our confidence in the soundness of our approaches.

In the following sections, we first evaluate each of the three main strategies with different limits and transfer functions (in case of the path- and ARG-based algorithms) and compare them to predicate and explicit analyses. Then we take the best k value and transfer function for each strategy and compare them to each other. Finally, we also present a summarizing table for all 22 configurations.

[4]These programs are automatically converted to CFA with Theta's C frontend [32].

4.3 Evaluate different limits for each strategy

In this section we examine the performance of every product abstraction strategy with four different limits. Besides that, they are also compared to PRED and EXPL.

4.3.1 Single state-based strategy

Table 4.1 shows the results of evaluating models with PROD2_STATE with the different limits. In case of this strategy, there is only one transfer function (which enumerates the possible states), therefore there are only configurations with the letter "E". The first column shows the configuration, the second represents the number of successful results (i.e., the algorithm terminated) out of the 430 total models and the third is the total execution time (e.g. sum of the time of the successful runs) in milliseconds. The different configurations are ranked from best to worst. We can see that the PROD2_STATE strategy preforms better with every limit than the PRED and EXPL algorithms. PROD2_STATE_E_01 has by far the best results with 354 verified models. We can see that effectiveness decreases as the limit increases. Although the difference is small for $k = 2, 8, 32$ (only 1 model).

Configuration	Succ. count	Total time (s)
PROD2_STATE_E_01	354	3852
PROD2_STATE_E_02	339	3564
PROD2_STATE_E_32	338	4178
PROD2_STATE_E_08	337	3983
PRED	325	7280
EXPL	312	2800

Table 4.1: Evaluate different k values for STATE.

In Figure 4.1, a heatmap can be seen representing the success rate and total time of the configurations in every category. The greener the tile is, the better the performance. EXPL has the best results in categories eca and ssh, although it has the worst overall performance because of the weak results in category plc. PRED has a good performance in every category except eca and ssh. Since the PROD2_STATE strategy has the same great performances in plc as PRED and in eca as EXPL, it could outperform both algorithms. The strategy with $k = 1$ is more successful than other k values due to the plc models.

Figure 4.2 represents a quantile plot [12]. It is comparing the maximal time per model on the vertical axis to the number of verified models on the horizontal axis. The performance of the configurations are represented by different coloured lines. A point (x, y) for a given configuration means that it could solve x models within y milliseconds for each. Lines shifted to the left could solve more models and lines shifted to the bottom require less time. Therefore, the line located closest to the bottom right corner yields the best performance. This figure shows essentially the same results as the heatmap in Figure 4.1: PROD2_STATE_E_01 is the most efficient strategy and PRED and EXPL are the worst, verifying a low number of models with a rather long execution time.

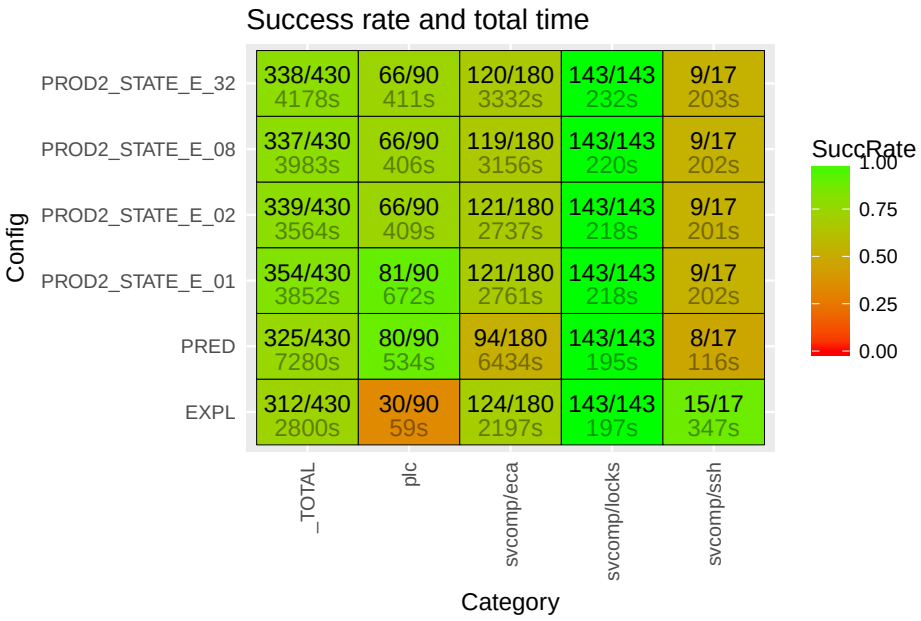


Figure 4.1: Heatmap of the PROD2_STATE strategies.

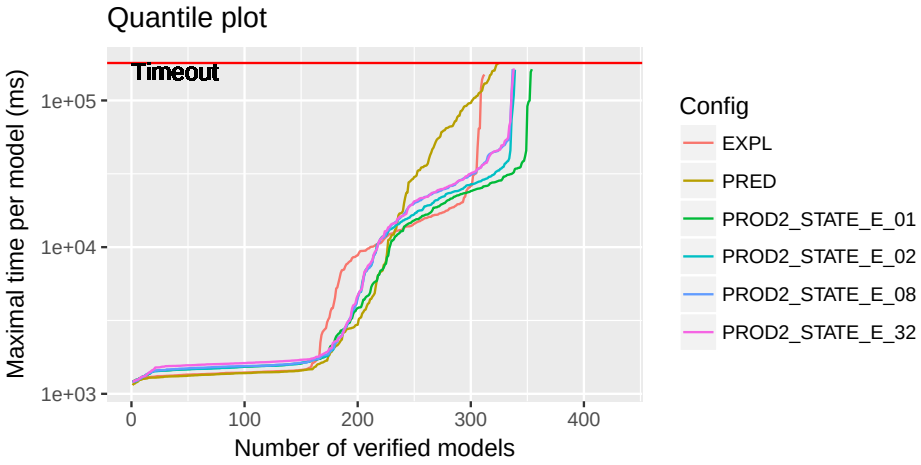


Figure 4.2: Quantile plot of PROD2_STATE.

4.3.2 Path-based strategy

The results of the measurements run with configurations PROD2_PATH, PRED and EXPL can be seen in Table 4.2. We can see, that the PATH strategy with the enumerating transfer function has better results than PRED and EXPL with every k value. The best configuration is the one where $k = 8$, verifying 338 models successfully. The second best with $k = 32$ verified the same amount of programs but with a greater total execution time. The PROD2_PATH_T configuration had the worst results verifying almost only half the models as the enumerating strategies and the base algorithms did. With limits 32 and 8, PATH had the same results, verifying 172 models. But with a decreasing limit, the performance also drops. PATH could only verify 97 models with limit 1, resulting in the worst overall performance.

Figure 4.3 shows a heatmap representing the success rate and total time of the PROD2_PATH strategy and PRED and EXPL algorithms in the different program cat-

Configuration	Succ. count	Total time (s)
PROD2_PATH_E_08	338	4057
PROD2_PATH_E_32	338	4085
PROD2_PATH_E_01	330	8937
PROD2_PATH_E_02	328	6406
PRED	325	7280
EXPL	312	2800
PROD2_PATH_T_32	172	3536
PROD2_PATH_T_08	172	3542
PROD2_PATH_T_02	135	4180
PROD2_PATH_T_01	97	2087

Table 4.2: Evaluate different k values for PATH.

egories. There is not one category where PROD2_PATH has the best results, but with the enumerating transfer function, it successfully combined the strengths of the PRED and EXPL algorithms, resulting in the best overall performance. The non-enumerating strategy had by far the worst performance in categories locks and ssh. In locks, it could only verify 4 models while every other configuration verified all 143 models successfully. PROD2_PATH_T_01 performed especially bad in the eca category, resulting in only 29 verified models from 180.

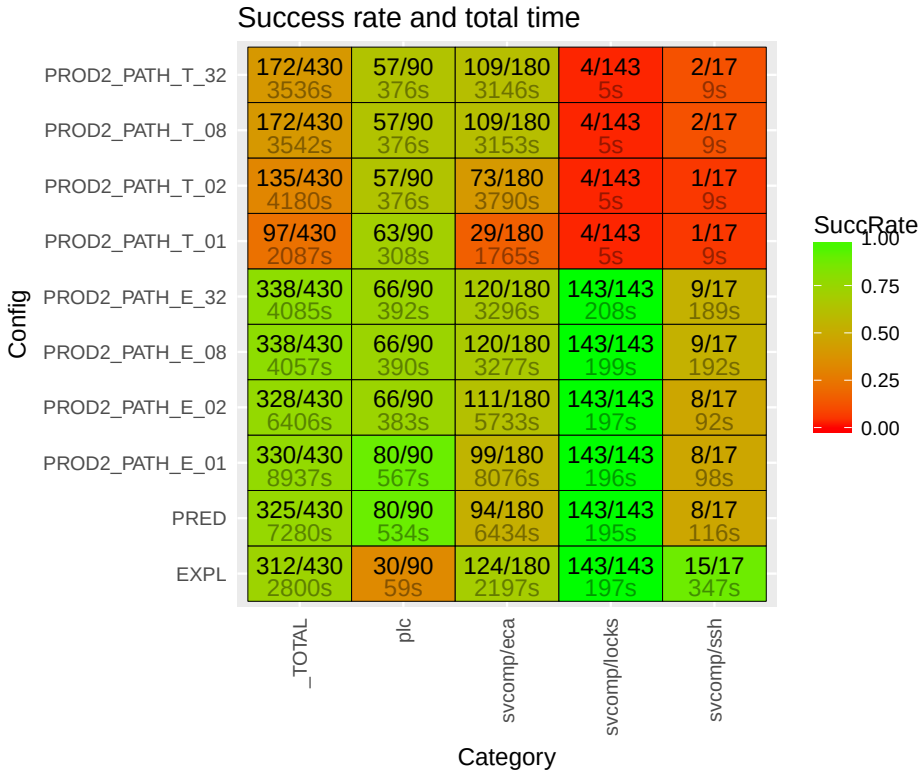


Figure 4.3: Heatmap of the PROD2_PATH strategies.

In Figure 4.6 the quantile plot representing the number of verified models and the maximal time per model for the PATH strategies can be seen. It shows that PROD2_PATH_E has the best performance followed by PRED and EXPL. The non-

enumerating PROD2_PATH_T strategies preform way worse than the others. We can also clearly see that the results of PATH get worse with decreasing the limit.

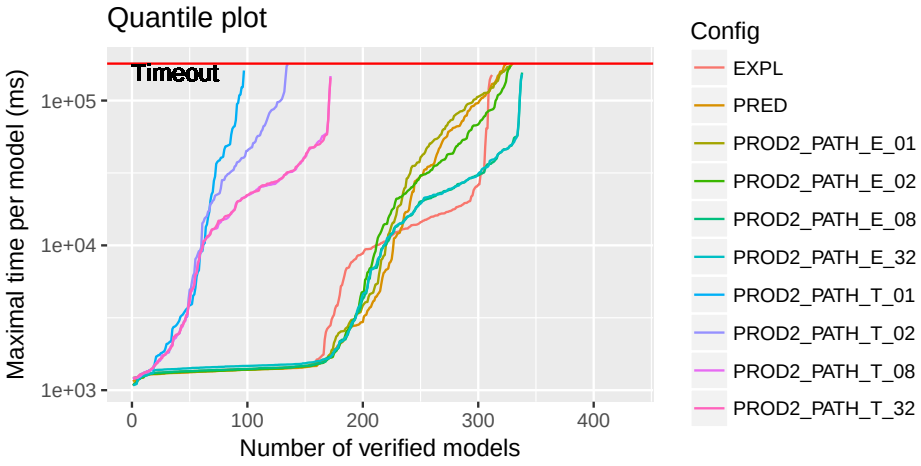


Figure 4.4: Quantile plot of PROD2_PATH.

4.3.3 ARG-based strategy

The results of evaluating PROD2_ARG can be seen in Table 4.3. The enumerating strategy performed better with all four limits than the PRED and EXPL algorithms. We can see, that bigger limits yield better performance, resulting in PROD2_ARG_E_32 being the most effective strategy, verifying 338 models. There is not much difference between the number of models verified, but the execution times vary greatly. PROD2_ARG_E_01 not only verified the least models of the enumerating strategies, but it took more than twice as long. The non-enumerating strategies all performed worse than the other algorithms. With limits 8 and 32, it successfully verified 172 models, with a very insignificant time difference. The PROD2_ARG_T_01 configuration had the worst overall performance, verifying only 98 models.

Config	Succ. count	Total time (s)
PROD2_ARG_E_32	338	4170
PROD2_ARG_E_08	337	3959
PROD2_ARG_E_02	334	5575
PROD2_ARG_E_01	327	8599
PRED	325	7280
EXPL	312	2800
PROD2_ARG_T_08	172	3352
PROD2_ARG_T_32	172	3358
PROD2_ARG_T_02	135	3891
PROD2_ARG_T_01	98	2107

Table 4.3: Evaluate different k values for ARG.

In Figure 4.5 we can see a heatmap representing the success rate and total time of the PROD2_ARG strategies for every program category. The enumerating ARG strategy has good results in every category. It is interesting to note that in the plc category, PROD2_ARG_E_01 verified remarkably more models, but because of the bad results in category eca, this configuration has the worst performance. The non-enumerating strategy

has the worst results in almost every category with every limit. It performed especially bad in the locks and ssh categories, yielding a very similar result to the results of the path-based strategy.

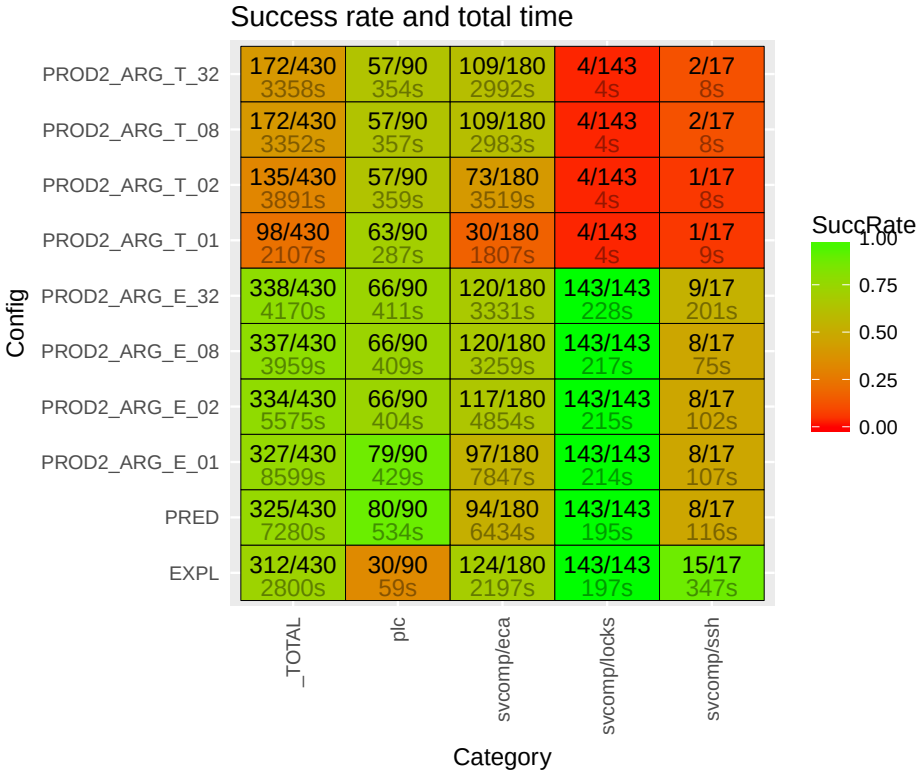


Figure 4.5: Heatmap of the PROD2_ARG strategies.

Figure 4.6 represents the quantile plot for PROD2_ARG strategies. Interestingly, there is a part in the plot where EXPL is below all the other algorithms, which means that it had a better performance. But later it could not solve as much models as the others, therefore it is not as efficient. This plot is also very similar to the quantile plot of the path-based strategy. The ARG-based strategy's efficiency is also decreasing with a lower limit.

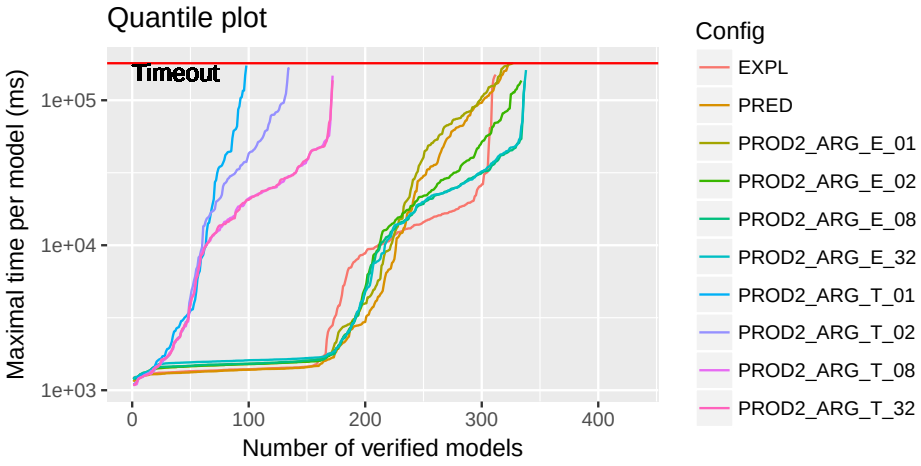


Figure 4.6: Quantile plot of PROD2_ARG.

4.4 Compare the best strategies

In this section we compare the configuration with best k value of each strategy with the PRED and EXPL algorithms. These are respectively the PROD2_STATE_E_01, the PROD2_PATH_E_8 and the PROD2_ARG_E_32 configurations. We can see, that with every strategy, the enumerating version yielded better results.

Table 4.4 presents the results of the best strategies. It shows, that PROD2_STATE_E_01 has the best results overall. It verified 354 models in less time, than the second best PROD2_PATH_E_08, which verified 338. The PROD2_ARG_E_32 verified the same amount of models successfully as the best path-based, but with a little longer execution time. All three strategies preformed better with their best configurations than the PRED and EXPL algorithms. It is also important to note that the execution time of the PRED algorithm is almost twice as long as the other configurations'.

Configuration	Succ. count	Total time (s)
PROD2_STATE_E_01	354	3852
PROD2_PATH_E_08	338	4057
PROD2_ARG_E_32	338	4170
PRED	325	7280
EXPL	312	2800

Table 4.4: Comparing the strategies with their best k value.

The heatmap of the best configurations can be seen in Figure 4.7. In the category locks, every strategy and algorithm successfully verified all the models. PROD2_STATE_01 has a an overall good performance in every category. PRED and EXPL also have good results, but there is one category for each of them, where their performance is really weak (eca for PRED and plc for EXPL). It can be clearly seen that all of our new strategies can successfully combine the advantages of explicit value-analysis and predicate abstraction to give an overall better performance.

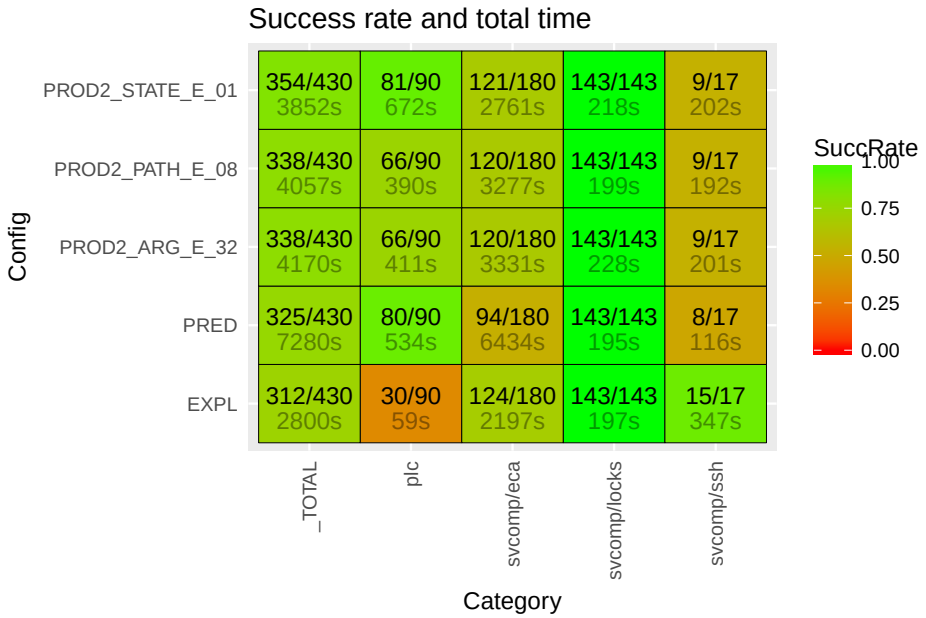


Figure 4.7: Heatmap of the best strategies.

The quantile plot of the best strategies can be seen in Figure 4.8. We can see that the algorithm with the longest runtime is by far the PRED. PRED and EXPL started well, but they verified less models than the product-based algorithms. This plot clearly shows that the most efficient strategy is the state-based, verifying the most models.

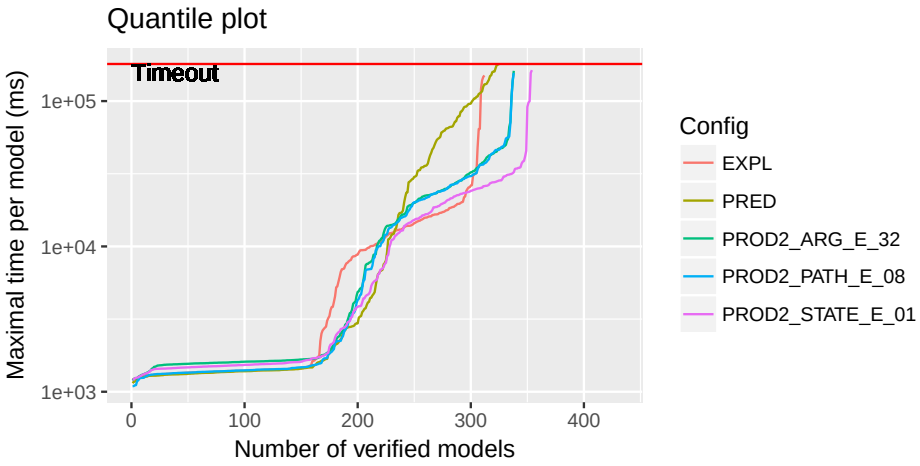


Figure 4.8: Quantile plot of the best strategies.

4.5 Compare all strategies with each other

Table 4.5 summarizes the results of every configuration starting from the best to the worst. This table contains all information from the previous tables. It shows that all enumerating strategies had a better performance than the PRED and EXPL algorithms, while the non-enumerating strategies preformed worse. PROD2_STATE_01 produced by far the best results with 354 successfully evaluated models with a good total execution time, followed closely by the other enumerating strategies with pretty similar results.

From these results we can conclude that the new product-based strategies with enumeration can successfully combine the strengths of explicit-value analysis and predicate abstraction in order to give an overall better performance.

Configuration	Succ. count	Total time (s)
PROD2_STATE_E_01	354	3852
PROD2_STATE_E_02	339	3564
PROD2_PATH_E_08	338	4057
PROD2_PATH_E_32	338	4085
PROD2_ARG_E_32	338	4170
PROD2_STATE_E_32	338	4178
PROD2_ARG_E_08	337	3959
PROD2_STATE_E_08	337	3983
PROD2_ARG_E_02	334	5575
PROD2_PATH_E_01	330	8937
PROD2_PATH_E_02	328	6406
PROD2_ARG_E_01	327	8599
PRED	325	7280
EXPL	312	2800
PROD2_ARG_T_32	172	3352
PROD2_ARG_T_32	172	3358
PROD2_PATH_T_32	172	3536
PROD2_PATH_T_08	172	3542
PROD2_ARG_T_32	135	3891
PROD2_PATH_T_02	135	4180
PROD2_ARG_T_32	98	2107
PROD2_PATH_T_01	97	2087

Table 4.5: Comparing all strategies.

Chapter 5

Conclusion

In our work, we presented two different CEGAR-based algorithms for software model checking, namely explicit-value analysis and predicate abstraction. Explicit-value analysis only tracks the values of a subset of program variables, while predicate abstraction focuses on tracking formulas over the variables. Both methods can be suitable for checking different kinds of software. In order to combine their advantages, we proposed a product abstraction domain with five different strategies. These approaches start by explicitly tracking each variable first and then switch to predicate abstraction, if the number of different values for a variable exceed a given limit. The difference between the methods is the way they count the values. Counting can be based on a single state, a path or the whole abstract reachability graph. For the path- and ARG-based strategies there is also another version, which uses unknown values instead of enumerating all the possible states.

We implemented our new strategies based on the open source Theta verification framework. We ran measurements on various input programs and compared the strategies to each other and the two basic algorithms (explicit values and predicates). We used benchmark models from the Software Verification Competition and industrial PLC codes from CERN. Measurements show, that the non-enumerating strategies are less efficient, but all the strategies which use the enumerating transfer function outperform pure explicit-value analysis and predicate abstraction. We can conclude that our new algorithms can successfully combine the advantages of the different abstract domains, providing a more efficient software model checking approach.

Future work. Even though the evaluation confirmed the efficiency of the new strategies, there are several opportunities to improve our work.

We could implement a strategy that first does not enumerate possible values for an explicitly tracked variable, but uses the unknown value. Then, if the refiner would add this variable again, we would start to enumerate values. Finally, if the number of the different values reach the limit, we switch to predicates.

It would be interesting to run the measurements on a wider set of models, possibly from different domains. This would help to generalize our results. Currently we only experimented with a few values for the limit. Evaluating more possibilities could give further insights. Furthermore, the CEGAR algorithm also has some other parameters (independent from the abstract domains), such as the search strategy in the abstract state space. It would be interesting to experiment with those parameters as well, to find a configuration that works the best with product abstraction.

Our evaluation at the moment is high level, because we only look at the summarised data. It would be interesting to look into the details to see why certain configurations have better or worse results in different program categories. This way we could identify possible improvements to our algorithms.

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